Résumés des Conférences Plénières

The Favard problem for multi-dimensional orthogonal polynomials.

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The Favard Lemma in the theory of orthogonal polynomials in one indeterminate is briefly recalled. After a short review of the several attempts to generalize this Lemma in the multi-dimensional case, the reasons why these attempts were not satisfactory will be explained.

Finally the solution of the problem, recently obtained in the joint paper :

Accardi L., Barhoumi A., Dhahri A. :
"Identification of the theory of orthogonal polynomials in d–indeterminates with the theory of 3–diagonal symmetric interacting Fock spaces on $\mathbb{C}_d$"
will be briefly described.

This solution suggests a new approach to the theory of orthogonal polynomials, that emphasizes the fact that it generalizes the usual mathematical structure of quantum mechanics and quantum field theory.

On Poincare's and Lions' lemmas and on De Rham's theorem

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We prove here the equivalence between many important properties concerning : the divergence operator, the Lions lemma, the Necas inequality, the Korn inequality and the weak lemma of Poincaré. Using then the Bogovskii operator and the Calderon-Zygmund theory, we give some isomorphism concerning the divergence operator. We give also a complete proof of the original De Rham theorem and we obtain some extension to the irrotational fields (see [1], [2]).

References

[1] Amrouche Chérif, Ciarlet Philippe G., Mardare Cristinel. Remarks on a lemma by Jacques-Louis Lions, C. R. Math. Acad. Sci. Paris, 352, 9, (2014), 691-695.
[2] Amrouche Chérif, Ciarlet Philippe G., Mardare Cristinel. On a lemma of Jacques-Louis Lions and its relation to other fundamental results, J. Math. Pures Appl. (9), 104, (2015), no. 2, 207-226.

This is a joint work with Ciarlet Philippe and Mardare Cristinel.

Pointwise estimates for the Ground states of Dirichlet fractional Laplacian with Hardy potentials

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On considère l'opérateur du type :

$$L_c = (-\Delta)^{\alpha/2}|_\Omega - c|x|^{-\alpha},$$

où $(-\Delta)^{\alpha/2}|_\Omega$ est le Laplacien fractionnaire défini sur un domaine borné, ouvert et régulier Ω de $\mathbb{R}^d$ contenant l'origine avec la condition zéro à l'extérieur et $0 < c \leq c^* := \frac{2^\alpha \Gamma^2(\frac{d+\alpha}{4})}{\Gamma^2(\frac{d-\alpha}{4})}$. Dans ce travail on établit des estimations ponctuelles pour la première fonction propre associée à L_c.
On montre en premier lieu que pour tout $c \in (0, c^*]$ il existe β pour lequel la fonction $w = |x|^{-\beta}$ vérifie

$$(-\Delta)^{\alpha/2} w - c|x|^{-\alpha} w = 0, \quad \text{au sens des distributions.}$$

Soit φ_0^c la fonction propre associée à λ_0^c vérifiant

$$(-\Delta)^{\alpha/2} \varphi_0^c - c|x|^{-\alpha} \varphi_0^c = \lambda_0^c \varphi_0^c.$$

On montre principalement qu'il existe $C_c > 0,\ C_c^1 > 0$ telles que

$$C_c w \delta^{\frac{\alpha}{2}} \leq \varphi_0^c \leq C_c w \delta^{\frac{\alpha}{2}}, \quad \text{pp}$$

où $\delta(x) = dist(x, \Omega^c)$.
Pour montrer ce résultat, on distingue le cas $c \in (0, c^*)$ et le cas $c = c^*$.
Notons par $T_t^c = e^{-tL_c}$, $t > 0$, $c \in (0, c^*]$. On montre que pour chaque T_t^c est ultracontractif et que

$$p_t(x,y) \sim e^{-\lambda_0 t} \delta^{\frac{\alpha}{2}}(x) \delta^{\frac{\alpha}{2}}(y) w(x) w(y) \text{ for large } t$$

et

$$G(x,y) \geq C \delta^{\frac{\alpha}{2}}(x) \delta^{\frac{\alpha}{2}}(y) w(x) w(y).$$

Two inverse problems defined by partially overdetermined boundary conditions in linear elasticity

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In spite of a great amount of work treating of the numerical resolution of Cauchy problems, very few publications are devoted to the sub-Cauchy elasticity system. The first part of this contribution is devoted to the sub-Cauchy problem in linear elasticity. Solving such a problem may be formulated as follows (in 2D situations) : Given the displacement field and ?one component of the traction on a given part of the boundary of an elastic body, reconstruct the displacement field in all the domain. An iterative method, borrowed from the domain decomposition community is proposed to solve ?such an inverse problem. Numerical results highlight the efficiency of the proposed method. The second part of the talk focuses on the analysis of the inverse geometrical problem of void detection defined by partially overdetermined boundary data. The approach proposed here is based on the minimization of constitutive law error functional. The topological asymptotic ?expansion of this functional is obtained. Numerical trials are also performed.

A Paley-Wiener theorem about the spectral parameter and a support theorem for general types of Dunkl spherical means

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The inversion formula for the Dunkl transform in polar coordinates is

$$f(x) = \int_0^\infty f_k^s(x)\, ds, \quad f \in \mathcal{D}(\mathbb{R}^d),$$

where f_k^s are "projections" of f into the eigenspaces of the Dunkl Laplacian Δ_k corresponding to the eigenvalue $-s^2$. Here the parameter k comes from Dunkl's theory of differential-difference operators. The natural question is how the properties of the function f relate to the properties of f_k^s? In this talk we will give an answer to this question whenever $supp(f) \subset \overline{B(O,R)}$. As a first application of this Paley-Wiener type theorem, we provide a spectral version of de Jeu's Paley-Wiener theorem for the Dunkl transform. The second application concerns a support theorem for general types of Dunkl spherical means.

Connection and Linearization Problems.

Several Approaches.

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Given two polynomial sets $\{S_n\}_{n\geq 0}$ and $\{P_n\}_{n\geq 0}$. The so-called *connection problem* between them asks to find the coefficients $C_m(n)$ in the expression :

$$S_n(x) = \sum_{m=0}^{n} C_m(n) P_m(x). \tag{0.1}$$

If $S_{i+j}(x) = Q_i(x)R_j(x)$, the relation (0.1) is known as *the general linearization relation.*
If $P_n = Q_n = R_n$, this problem is reduced to the *standard linearization* or *Clebsch-Gordan type problem.*
The study of the connection and linearization problems has gained an increasing interest. A wide variety of methods, based on specific properties of the involved polynomials, has been devised for computing the connection and linearization coefficients either in explicit form or by means of recursive relations.

In this talk, we give a survey of several techniques used to solve the connection and the linearization problems.

References and Literature for Further Reading

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Stabilisation des graphes

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La stabilisation des systèmes d'ondes sur les graphes soumis à un seul feedback, dépend des propriétés arithmétiques des rapports des longueurs de ses tiges. Nous présentons dans cet exposé quelques résultats de stabilisation des arbres non dégénérés et nous donnons les meilleurs taux de décroissance de l'énergie (ou de l'énergie local) dans le cas de quelques arbres dégénérés (borné ou non).

Uniqueness and stability results in some inverse spectral problems.

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When $\Omega \subset \mathbb{R}^N$ is a bounded Lipschitz domain and $a, \rho \in L^\infty(\Omega)$ are three functions satisfying $\min(a, \rho) \geq \epsilon_0$, for some fixed $\epsilon_0 > 0$ and $q \in L^\infty(\Omega)$, one considers the linear elliptic operator

$$u \mapsto Lu := -\mathrm{div}(a\nabla u) + qu$$

under various boundary conditions (for instance Neumann boundary condition). We associate to this operator its *boundary spectral data*, that is the set

$$\mathrm{BSD}(a, \rho, q) := \{(\lambda_k, \gamma_0(\varphi_k))\ ;\ k \geq 1\}$$

where the eigenvalues λ_k and the eigenfunctions φ_k are given by

$$L\varphi_k = \lambda_k \rho \varphi_k, \quad (a\nabla\varphi_k)\cdot \mathbf{n} = 0 \ \text{ on } \ \partial\Omega, \quad \int_\Omega \varphi_k(x)\varphi_j(x)\,\rho(x)dx = \delta_{kj},$$

and $\varphi \mapsto \gamma_0(\varphi) := \varphi_{|\partial\Omega}$ denotes the trace operator on the boundary $\partial\Omega$.

We shall give a short review of results pertaining to the question which consists in the determination of either of the coefficients a, ρ, a through the knowledge of the boundary spectral data $\mathrm{BSD}(a, \rho, q)$.

The case of a waveguide, where $\Omega := \omega \times \mathbb{R}$ with $\omega \subset \mathbb{R}^2$ a bounded Lipschitz domain, while $a \equiv \rho \equiv 1$ and q is periodic in the direction x_3, will be the main subject of our presentation.

Deforming discontinuous subgroups for nilpotent homogeneous spaces.

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Let G be the $(n+1)$-dimensional threadlike Lie group, H an arbitrary closed connected Lie subgroup of G and $\mathbb{G} \subset G$ an abelian discontinuous subgroup for G/H. We present in this talk some topological properties of the parameter space $\Gamma(\mathbb{G}, G, H)$ and the deformation space $\mathcal{T}(\mathbb{G}, G, H)$. Writing explicitly a layering of these spaces, we establish a stability on layers theorem and we show that the local rigidity holds if and only if Γ is finite.

Structures modérées en topologie, géométrie et théorie des nombres.

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A l'origine, les structures modérées (géométrie o-minimale) ont constitué un cadre général permettant d'exclure certains objets "pathologiques" et de disposer d'un formalisme agréable et flexible dans lequel les objets ont des propriétés topologiques et géométriques raisonnables. Plus récemment elles ont permis d'effectuer des avancées spectaculaires en théorie des nombres. Nous présenterons un panorama général de ces questions, en mettant l'accent principal sur les structures réelles tout en mentionnant des progrès récents dans d'autres contextes comme celui de la géométrie non-archimédienne.

Les algèbres n-aires : de la physique aux mathématiques

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Les algèbres de Lie et les algèbres de Poisson jouent un rôle fondamental en mathématique et en physique. Leurs généralisations, connu sous le nom d'algèbres n-Lie ou de Nambu sont aussi apparues naturellement en physique dans plusieurs contextes. La mécanique de Nambu qui est une généralisation de la mécanique hamiltonienne utilise des structures ternaires ou n-aires, cela permet de construire une théorie avec plusieurs hamiltoniens. Par ailleurs, les algèbres n-aires servent à construire des solutions de l'équation de Yang-Baxter qui est apparue d'abord en mécanique statistique. L'objectif de cette exposé est de rappeler les bases de la théorie et de donner un aperçu des développements récents de ces structures.

Recent progress in conjugacies between circle homeomorphisms with breaks

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In our previous project (Ergodic Theory and Dynamical Systems, 2014), we showed that the conjugacy, from a P-circle homeomorphism f with several break points, having irrational rotation number α and Df absolutely continuous on every continuity interval of Df, to the rotation R_α is a singular homeomorphism, provided that the total product of f-jumps is non trivial. The propose of this talk is to discuss a joint work with Abdelhamid Adouani (arXiv :

1512.03327v1, 2015) where we expand this theory to the case of conjugacies between two P-circle homeomorphisms having irrational rotation numbers of bounded type.

M-Essential Spectra of 2×2 Matrix Operator and Application to Transport Operator

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In this talk we give some results on perturbation theory of 2×2 block operator matrices on the product of Banach spaces. Furthermore, we investigate their M-essential spectra. Finally, we apply the obtained results to determine the M-essential spectra of two group transport operators with general boundary conditions in the Banach space.

Coherent states and Berezin transforms attached to Landau levels

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We review the definition and properties of coherent states with examples. We construct coherent states attached to Landau levels (Poincaré disk, Euclidean plane and the Riemann sphere). We define their corresponding coherent states transforms and we obtain characterization theorems for spaces of bound states of the particle. Generalization to $\mathbb{C}^n$ and to the complex unit ball $\mathbb{B}^n$ are also discussed. In these two cases, we apply a coherent states quantization method to recover the corresponding Berezin transforms and we give formulae representing these transforms as functions of Laplace-Beltrami operators.

Variational Analysis of Some Classes of Equilibrium Problems

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We report on recent results in collaboration with Boualem Alleche concerning the qualitative analysis of equilibrium problems. We establish several abstract variational principles in a general setting, in particular a version of the Ekeland variational principle for countable systems of nonconvex equilibrium problems. Applications include existence results for quasi-hemivariational inequality problems and fixed point properties of multi-valued operators.

Dérivations et systèmes intégrables

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Le cadre naturel des systèmes intégrables de dimension infinie est un formalisme algébrico-géométrique d'algèbres de Lie de champs de vecteurs, celles-ci étant elles-mêmes engendrées par des dérivations. Le cas le plus connu est celui de l'algèbre de Virasoro et de ses "prolongements" dites algèbres de Gelfand-Dikii ou algèbres-W. Ces constructions admettent des généralisations naturelles au cas superalgèbre/supergéométrie, si la dérivation est paire . Le cas où les dérivations sont impaires produit des structures algébriques toutes différentes, mais à partir desquelles on peut parfois construire des systèmes d'équations intégrables.

On some differential operators on weighted graphs

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After a brief survey on finite graphs, we define some differential operators on a weighted infinite locally finite graph by the data of some weights on the vertices and some weights on the edges. We will concentrate here on one or second order differential operators like Gauß-Bonnet, Laplace, Schrödinger operators.
Then we discuss essential self-adjointness of such operators in special cases with geometric conditions.

On Quasi-Frobenius Rings

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A two-sided artinian ring R is called *Quasi-Frobenius* (*QF-ring*) if it satisfies the following annihilator equations :

$$\begin{aligned} r_R l_R(T) &= T, \text{ for all right ideals } T \text{ of } R_R \\ l_R r_R(L) &= L, \text{ for all left ideals } L \text{ of } R_R. \end{aligned}$$

Where the left annihilator of a right ideal T of R is defined by $l_R(T) = \{x \in R : xT = 0\}$, and the right annihilator of the left ideal L of R is defined in a similar way.

The class of Quasi-Frobenius rings is one of the most important classes of non-semisimple artinian rings, and in the early historical developments they were considered first by T. Nakayama in the 1940's in the representation theory of group algebras of finite groups.

Equivalently, a ring R is *QF-ring* if R satisfies any of the following conditions :

(1) *R is left (or right) artinian and if $\{e_1, e_2, \cdots, e_n\}$ is a basic set of primitive idempotents of R, then there exists a (Nakayama) permutation σ of $\{1, 2, \cdots, n\}$ such that $soc(Re_k) \cong Re_{\sigma k}/Je_{\sigma k}$ and $soc(e_{\sigma k}R) \cong e_k R/e_k J$.*
(2) *R is left (or right) perfect and left and right selfinjective.*
(3) *Every left (or right) R-module embeds in a free module.*
(4) *R is left (or right) noetherian and every one-sided ideal of R is an annihilator.*

The equivalence between (1) and (4) is essentially due to T. Nakayama , (2) is due to B. Osofsky and (3) is due to C. Faith and E. Walker.

There are numerous other equivalent conditions that a ring is QF ; we have chosen these because of their relevance to three problems of Faith which we will refer to as follows :

The Faith problem : Is every left (or right) perfect, right selfinjective ring QF ?

The FGF-problem : Is every right *FGF*-ring QF ? (A ring is called a right *FGF-ring* if every finitely generated right module embeds in a free module.)

The Faith-Menal problem : Every strongly right Johns ring is QF. (A ring R is called *right Johns* if R is right noetherian and every right ideal is an annihilator ; and R is called *strongly right Johns* if the matrix ring $M_n(R)$ is right Johns for all $n \geq 1$.)

In this talk we review some recent work on these problems.